An Interferometer for Direct Recording of Refractive Index Distributions

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The well established principle of division of amplitude in two-beam interference is used to locate constant optical thicknesses in an experiment where the refractive index is a continuously varying function of position. This is applied to the experimental situation of determining the temperature distribution around a plane heat source in a transparent liquid. It turns out that it is most convenient to study the interference in the light reflected by the front and rear windows of the cell. The highly coherent light from a helium-neon gas laser is a basic requirement for this interferometry, where one is using optical path differences of the order of half a meter. The focusing of the optical system is discussed in some detail. A second order aberration theory gives the condition for the elimination of what corresponds to the Wiener skewness.

Among the different experimental techniques of studying transport processes such as diffusion, thermal diffusion, thermal conductivity etc. optical techniques have been widely accepted as convenient and reliable methods. When high sensitivities are required, it is necessary to use an optical system resembling the one described by PHILPOT and COOK [1], who are using a modified Rayleigh interferometer. Similar fringes, which give a direct plot of the refractive index distribution, can also be obtained with the Mach–Zehnder interferometer. The main experimental inconvenience in these cases is, however, that one has to make arrangements not only for the light beam through the test region but also for the reference beam, which is necessary when recording a complete distribution. The problem is thus to make an optical arangement which automatically introduces a reference beam.

It is obvious that a very simple way is the use of so-called Fizeau fringes [2]. These fringes, which follow lines of equal optical thickness, are used in the Fizeau interferometer, where the film thickness generally is of the order of a couple of wave lengths. In a few cases it has been possible to construct interferometers using Fizeau fringes with films which are not thin. In these cases, however, there are so severe restrictions on the size of a conventional light source, in order to get a sufficient degree of coherence, that the interferometer would be impracticable when recording a transient phenomenon that limits the exposure time. Lately the situation has, however, completely changed by the rapid development of continuous gas lasers, which have turned out to be excellent light sources with practically no restriction due to coherence. Thus the film of a Fizeau interferometer could in our case be replaced by a cell containing a liquid with an immersed "hot foil" and the interference between the light reflected at the entrance and exit windows could be studied.

The details of the optical arrangement are shown in Fig. 1. The two lenses L_1 and L_2 are used to enlarge the beam from the laser L and to give parallel light in the cell. L_3 is the camera lens, which forms an image on the photographic film (C) of a particular

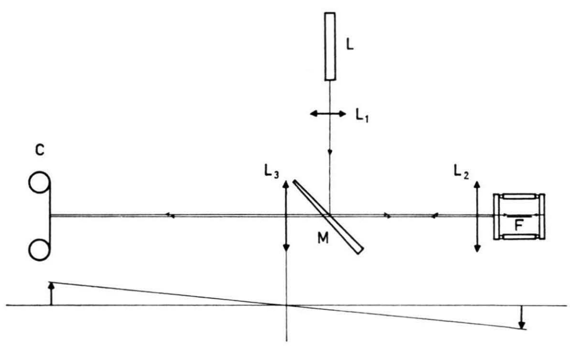


Fig. 1.

plane in the test region (F). The half silvered wedge (M) is introduced in order to get a perfectly reversible light path through the cell and thereby facilitate the calculation of the optical path difference introduced by the temperature increase in the liquid. From an experimental point of view it is most favourable to adjust the cell windows to form a very small angle, which gives rise to a basic fringe pattern in the image. The deviation of a particular fringe is then a measure of the additional optical path introduced by a change of the refractive index somewhere in the test region. To get a suitable distance between the fringes the angle between the windows must be kept very small. As a matter of fact it should be about thirty seconds. With such a small angle it is evident that this does not influence the computation of the optical path at all.

[1] J. ST. L. PHILPOT and G. H. COOK, Research, London **1**, 234 [1948].

[2] M. BORN and E. WOLF, Principles of Optics, Pergamon Press, 3rd ed., London 1965, p. 286–291.

In this kind of interferometry with long path difference, being in our case slightly less than half a meter, the only possible light source is a laser operating substantially in one single axial mode. This is important in order to avoid periodic nulls in the interference pattern. A single mode may be obtained by passing the laser radiation through a Fabry-Perot interferometer tuned to a particular mode of the laser or by operating the laser near threshold, which proved to be satisfactory in our investigation. The laser used in this work had a mirror spacing of 30 cm, which means that only three axial modes can be in simultaneous oscillation under high gain conditions in the plasma tube.

The entrance and exit windows of the cell would form an ideal Fabry-Perot resonator if the reflectivity were high enough. We are, however, using windows of optical quality glass with a reflectivity of about four percent, which means that multiple reflections are not contributing to the interference pattern.

Theory

Since the deviation of one particular fringe is a measure of the optical path introduced by the variation of the refractive index in the cell, it is important to consider the calculation of the optical path in some detail. In a first order approximation we can take twice the value given by Eq. (8) in an earlier paper [3]. The optical path is, however, dependent upon how the optical system is focused. In order to reveal such details it is necessary to compute the second order aberrations when the light traverses a test region with a general refractive index distribution. SVENSSON has given a very complete description of the aberrations in interferometric measurements of concentration gradients [4]. His results are, however, not applicable to our experimental arrangement because his reference beam is assumed to pass through a reference cell while the reference beam in our case is reflected at the first surface of the entrance window and never passes the test region.

In the following computations we are using a similar procedure as SVENSSON, but our calculations become more complex, because of the particular optical arrangement. When considering the refractive index distribution around a hot foil suspended in a liquid the situation is becoming even more complex. In order to simplify the problem we assume a one dimensional refractive index distribution in the

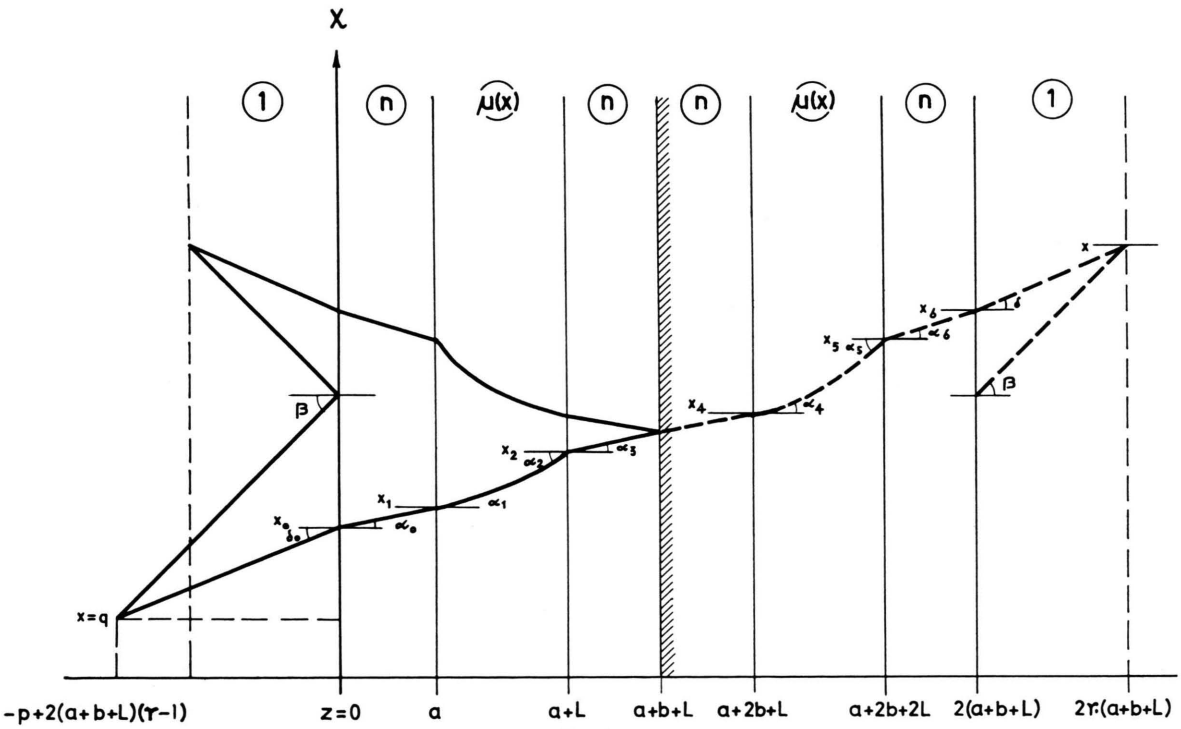


Fig. 2.

[3] S. E. GUSTAFSSON, Z. Naturforsch. 22 a, 1005 [1967]

[4] H. SVENSSON, Optica Acta 1, 25 [1954]; 3, 164 [1956].

middle of the cell and neglect the difference of the refractive index of the water and the glass. The last assumption is discussed by SVENSSON and can easily be corrected for. As to the details of the computations we refer to the papers by SVENSSON.

In Fig. 2 the z-axis is the optical axis and the cell is assumed to be situated between

$$z=0 \quad \text{and} \quad z=a+b+L\,.$$

After reflexion at the surface $z=a+b+L$ the light beam returns through the cell and the interference is supposed to take place in the plane

$$z=-2(a+b+L)(r-1)\,.$$

This assumption is correct if we are using a perfect optical system. In order to get similarity with SVENSSON's approach we study the mirror image of the optical path as indicated in Fig. 2. With the notations of Fig. 2 it is easy to write down an exact expression of the optical path.

$$\begin{aligned}\Delta S=&[p-2(a+b+L)(r-1)]/\cos\delta_0+a\,n/\cos\alpha_0\\&+\int_a^{a+L}[\mu(x)/\cos\alpha]\,\mathrm{d}z+2\,b\,n/\cos\alpha_3\\&+\int_{a+2b+L}^{a+2b+2L}[\mu(x)/\cos\alpha]\,\mathrm{d}z+a\,n/\cos\alpha_6\\&+2(r-1)(a+b+L)/\cos\delta-p/\cos\beta\,.\end{aligned}$$

This expression is not practicable because it contains variables and unknown parameters. It must be transformed into a form containing only experimentally known parameters and the coordinate x of the plane brought into focus. By using a number of Taylor expansions ΔS can be converted into a power series of a, b, and L. The general principle is to express the optical path of one particular region in terms of the quantities at the beginning of the region in question. For instance, if we compute the optical path of a beam passing the region between the planes $z=a$ and $z=a+L$, it may be expressed in terms of the angle α_1, the coordinate x_1, and the distance L, giving

$$\int_a^{a+L}[\mu(x)/\cos\alpha]\,\mathrm{d}z=L\,\mu_1+\tfrac{1}{2}L\,\mu_1\,\alpha_1^2+L^2\,\mu_1'\,\alpha_1+\tfrac{1}{3}L^3\,\mu_1'^2/\mu_1\,,$$

where the index 1 means that the function should be calculated at x_1. A similar equation is obtained when computing the optical path from $z=a+2\,b+L$ to $z=a+2\,b+2\,L$, the only difference being that we get index 4 instead of 1. All the other parts of the total optical path difference (ΔS) may in principle be obtained by using a second order expansion of the secant function. The various angles compiled in the different expressions must then be transformed into a form containing only known parameters. This gives the following relations

$$\begin{aligned}\delta=n\,\alpha_6=\mu\,\alpha_5=\delta_0+2\,L\,\mu'\,,\\ \mu\,\alpha_4=n\,\alpha_3=\mu\,\alpha_2=\delta_0+L\,\mu'\,,\\ \mu\,\alpha_1=n\,\alpha_0=\delta_0=\beta\,,\end{aligned}$$

where the prime denotes a differentiation with respect to the coordinate x. The next step is to make a transformation between the different x_i ($i=0$, 1, 2, 4, 5, 6) and x, and thereby eliminate all these unknown variables.

$$\begin{aligned}x=&x_0+2\,\delta_0(a/n+b/n+L/\mu)\\&+2\,L\,\mu'(a/n+b/n+L/\mu)\\&+2(a+b+L)(r-1)(\delta_0+2\,L\,\mu')\,,\\ x_6=&x_0+2\,\delta_0(a/n+b/n+L/\mu)\\&+2\,L\,\mu'(a/n+b/n+L/\mu)\\=&x_5+(\delta_0+2\,L\,\mu')(a/n)\,,\\ x_4=&x_0+\delta_0(a/n+2\,b/n+L/\mu)\\&+L\,\mu'(2\,b/n+L/2\,\mu)\\=&x_2+(\delta_0+L\,\mu')(2\,b/n)\,,\\ x_1=&x_0+\delta_0(a/n)\,.\end{aligned}$$

In these computations we have assumed that it is possible to make a direct transformation to the image plane because both the angles and the coordinates appear only in the second order terms. The transformations are carried out according to the equation

$$\mu(x_i)=\mu(x)-(x-x_i)\,\mu'(x),\quad i=0,1,2,4,5,6.$$

The final expression of the optical path becomes

$$\begin{aligned}\Delta S=&2\,L\,\mu+2\,n(a+b)\\&-\delta_0^2[a/n+b/n+L/\mu+(a+b+L)(r-1)]\\&-2\,\delta_0\,L\,\mu'[a/n+b/n+L/\mu\\&\qquad+2(a+b+L)(r-1)]\\&-L^2\,\mu'^2[2\,a/n+b/n+4\,L/3\,\mu\\&\qquad+4(a+b+L)(r-1)]\,.\end{aligned}$$

The second order terms in this equation can now be made to disappear by a proper choice of the experimental conditions. If we use a collimated light beam parallel with the optical axis the entrance angle δ_0 vanishes and we are left with only one second order term. By a proper adjustment of r this last term, which corresponds to the Wiener skewness, can be made to disappear. The required value is

$$r = 1 - (a/2\,n + b/4\,n + L/3\,\mu)/(a + b + L)\,.$$

We can compare this r-value with the one obtained by SVENSSON by putting $a = b = 0$, which gives $r = 1 - 1/3\,\mu$. The main difference between the two r-values is that we get a dependence on the absolute value of the refractive index in the test region while SVENSSON does not. This seams very reasonable because our reference beam never enters the regions with a refractive index higher than unity.

In our experiments we used an optical arrangement with the following dimensions $a = 6.2$ cm, $b = 6.8$ cm, $L = 5.0$ cm and $n = \mu = 1.33$ giving $r = 0.73$.

Experimental Tests

The cell and support of the foil used in these experiments are described in an earlier paper [3]. The windows of the cell were placed between rubber rings in a holder connected to the cell wall with three adjustable screws, thereby providing a possibility to achieve the desired angle between the glass plates.

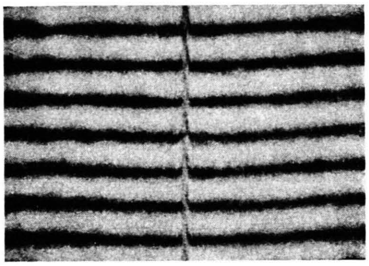
The deviation of a particular fringe (Fig. 3) is a measure of the additional optical path introduced by the variation of the refractive index around the heated foil and this may be expressed by the following equation [3].

$$Q_{\rm d}(\partial\mu/\partial T)\,\frac{2(\varkappa\,t)^{1/2}}{\Lambda}\,[\pi^{-1/2}\exp(-x^2/4\,\varkappa\,t) - [x/(4\,\varkappa\,t)^{1/2}]\cdot\mathrm{erfc}[x/(4\,\varkappa\,t)^{1/2}]] = y\,\lambda/m\,,$$

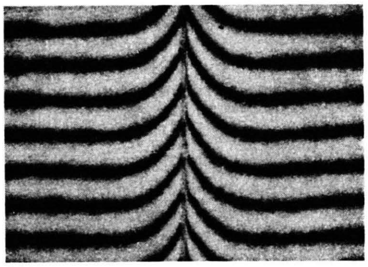
where y is the deviation of a fringe and m is the distance between any two of the parallel fringes corresponding to a path difference of one wave length λ. This equation can be used to determine the thermal conductivity Λ as soon as the thermal diffusivity $\varkappa$ ($= \Lambda/s$; s specific heat per unity of volume) is known. By measuring a pair of y-values and the corresponding x-coordinates at a particular time t, a parameter σ may be computed by iteration from the expression

$$y_2[\pi^{-1/2}\exp(-x_1^2/\sigma^2) - (x_1/\sigma)\cdot\mathrm{erfc}(x_1/\sigma)] = y_1[\pi^{-1/2}\exp(-x_2^2/\sigma^2) - (x_2/\sigma)\cdot\mathrm{erfc}(x_2/\sigma)]\,.$$

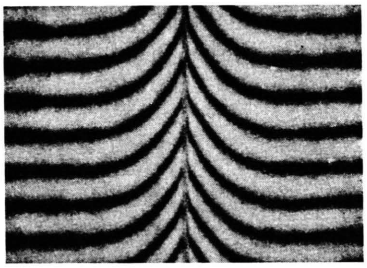
If $\sigma^2/4$ then is plotted versus t in a diagram, $\varkappa$ is obtained as the slope of the straight line. In this way it is not necessary to consider any zero time correction, which is important from experimental point of view. In our experiments we had the facilities to measure the output of power per unit length of the foil $Q_{\rm d}$ and we did not have to rely on measurements of the density and the specific heat.



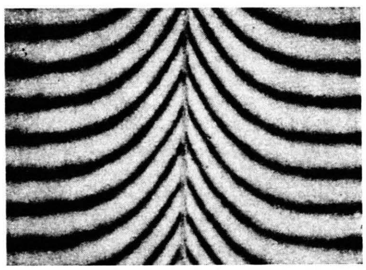
$t = 0$ sec



$t = 10$ sec



$t = 20$ sec



$t = 40$ sec

Fig. 3.

Results and Discussion

To test the utility of the instrument, we have determined the thermal conductivity and the thermal diffusivity of water at room temperature (Table 1).

Temp. °C	Power per unit length of the foil J/m · sec	Thermal diffusivity mm^2/sec	Thermal conductivity J/m · sec °C
23.40	4.114	0.1460	0.6067
23.60	4.130	0.1437	0.6111
23.35	4.225	0.1446	0.6008

Table 1. Experimental results obtained from measurement on water. The dimensions of the silver foil were $0.00980 \times 50.3 \times 188.1$ mm^3.

The most interesting thing is that we can work with a liberation of heat per unit length of the foil, which is at least ten times less than in the earlier experiments and still maintain the same accuracy. This is due to the higher sensitivity of the interferometer and has the immediate consequence that the temperature increase in the foil and its vicinity can be kept about ten times lower than before. The onset of convection is thus greatly delayed and we have not been able to visually detect any sign of convection before 90 seconds. This means that we have an available time scale, which is comparable with the one of the hot wire cell with electrical temperature recording [5]. The advantage of our technique is that it is possible to determine both the thermal conductivity and the thermal diffusivity in one single experiment, which is an excellent check of the internal consistency of the run.

In summary, this Fizeau fringe interferometer yields a precision which is comparable with any other optical instrument and has a sensitivity which exceeds that of similar interferometers. From the point of view of convenience it has the advantage of being easy to adjust. Moreover, this instrument should be very suitable for measuring diffusion and Soret coefficients, where it also is desirable to get a direct recording of the refractive index distribution. The condition for the proper focusing of the interferometer as treated in this paper should be applicable to all these cases by choosing the proper values of the constants in the final equations.

This work has been financially supported by the Swedish Council for Applied Research and by "Adlerbertska Forskningsfonden", which is gratefully acknowledged.

[5] J. K. Horrocks and E. McLaughlin, Proc. Roy. Soc. London A **273**, 259 [1963].